There are two ways we will use to solve rational equations. The first way is to get the same denominator on both sides of the equation.

<u>Example 1</u>- Solve $\frac{3x}{x+1}+\frac{6}{2x}=\frac{7}{x}$

Step 1- Get the denominators the same on both sides

We can actually reduce the second fraction, and equations becomes

$$\frac{3x}{x+1}+\frac{3}{x}=\frac{7}{x}$$

We need to make this equation look like

$$\frac{???}{x(x+1)}=\frac{???}{x(x+1)}$$

Let's multiply our fractions to make it happen!

$$\frac{3x}{x+1}\bullet\frac{x}{x}+\frac{3}{x}\bullet\frac{x+1}{x+1}=\frac{7}{x}\bullet\frac{x+1}{x+1}$$

$$\frac{3x^2}{x(x+1)}+\frac{3x+3}{x(x+1)}=\frac{7x+7}{x(x+1)}$$

$$\frac{3x^2}{x(x+1)}+\frac{3x+3}{x(x+1)}=\frac{7x+7}{x(x+1)}$$

$$\frac{3x^2+3x+3}{x(x+1)}=\frac{7x+7}{x(x+1)}$$

Denominators are now the same on both sides of the equation

Step 2- Drop the denominators

$$3x^2+3x+3=7x+7$$

Step 3- Solve

$$3x^2 + 3x + 3 = 7x + 7$$
$$3x^2 - 4x - 4 = 0$$
$$(3x + 2)(x - 2) = 0$$
$$3x + 2 = 0 \qquad x - 2 = 0$$
$$3x = -2 \qquad \boldsymbol{x = 2}$$
$$\boldsymbol{x = -\frac{2}{3}}$$

Step 4- Check solutions

Look at the denominators of the original problem. Are any of the solutions we got in Step 3 not allowed to be in the domain because they make the denominator equal 0?

We are OK. Our denominators were $x + 1$ and $x$. That means for the domain, $x \neq 1$ and $\neq 0$. Since our answers do not cause a problem, they are both allowed to remain as solutions. If a solution from Step 3 is not allowed in the domain, you have to eliminate it from your final answer.

$$\boxed{\boldsymbol{x = -\frac{2}{3}, 2}}$$

Another way to solve rational equations is to cross-multiply. This is possible when you have single fractions on both sides of the equation.

<u>Example 2</u>- Solve $\frac{x}{x^2-8} = \frac{2}{x}$

Step 1- Cross multiply

$$\frac{x}{x^2-8} = \frac{2}{x}$$

$$x \bullet x = (x^2 - 8) \bullet 2$$
$$x^2 = 2x^2 - 16$$

Step 2- Solve

$$x^2 = 2x^2 - 16$$
$$0 = x^2 - 16$$
$$0 = (x+4)(x-4)$$
$$x + 4 = 0 \quad x - 4 = 0$$
$$\boldsymbol{x = -4} \quad \boldsymbol{x = 4}$$

Step 3- Check solutions

Look at the denominators of the original problem. Are any of the solutions we got in Step 2 not allowed to be in the domain because they make the denominator equal 0?

We are OK. Our denominators were $(x^2 - 8)$ and $x$. That means for the domain, $x \neq \pm\sqrt{8}$ and $x \neq 0$. Since our answers do not cause a problem, they are both allowed to remain as solutions. If a solution from Step 2 is not allowed in the domain, you have to eliminate it from your final answer.

$$\boxed{\boldsymbol{x = \pm 4}}$$